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Transgressing personal foregrounds through the learning of mathematics

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ABSTRACT

According to humanistic psychology, human beings have the potential to shape and change their lives. People are self-determining creatures capable of making decisions about what their existence will be like and who they will become. We interpret this specifically human feature as an empowerment to design one's life and also to influence constructive changes in the world. Such a view of the human nature can facilitate teachers' efforts in shaping children's abilities to cope with difficulties and foster positive beliefs regarding self-esteem, self-efficacy and control over not only mathematical problems. We analyse personal foregrounds keeping in mind that our backgrounds unquestionably affect our future. Moreover, we show that from the early years of schooling, mathematical education can provide students with opportunities to transgress their personal foregrounds. Finally, we postulate that mathematical education may serve as a means for changing one's life.

ARTICLE HISTORY

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KEYWORDS

Foreground; mathematics education; transgressions in education; problem-solving

Introduction

In this article, we scrutinize the notion of a person's foreground understood, according to Skovsmose (2005), as 'the opportunities, which the social, political and cultural situation provides for this person' (p. 6). We argue that one's foreground can and should be shaped, transformed and transgressed by means of education and postulate that this process would begin at the early years of schooling. We particularly highlight the potential impact that mathematics education can have on person's foregrounding. We choose to focus on mathematics because it unquestionably permeates everyone's life and plays an important role throughout the lifespan. All over the world, mathematics instruction is an integral part of school curricula, starting from the stage of kindergarten education up to (at least) the mandatory secondary level. Furthermore, many fields of study, as well as different kinds of professions, require some mastery in mathematics. It is assumed that by learning mathematics, which begins with early experiences, young people obtain some general and specific knowledge, skills and competencies that enable them to become successful adults, confident when using math at work and in their daily lives. It is important to note, however, that these educational goals are based on the conviction that the skills trained at school are being transferred to the life of a person. Thus, the learning of mathematics, which takes place throughout most of the stages of education, is oriented not only towards upgrading and broadening one's mathematical skills, but also towards generalizing some mathematics-laden competencies to various domains of human life. Now, we argue that one of the most important competencies that can be transferred from the school setting into the domain of real life is that of problem-solving. In the last decades problem-

CONTACT Barbara Pieronkiewicz 🐼 barbara.pieronkiewicz@up.krakow.pl 🗈 Institute of Mathematics, Pedagogical University of Cracow, Podchorążych 2, St., Cracow 30-084, Poland © 2017 Informa UK Limited, trading as Taylor & Francis Group solving has earned a place in the overall educational goals, attracting especially the careful attention of mathematics and science educators. The problem-solving competency is usually defined as 'an individual's capacity to engage in cognitive processing to understand and resolve problem situations where a method of solution is not immediately obvious. It includes the willingness to engage with such situations in order to achieve one's potential as a constructive and reflective citizen' (OECD, 2013, p. 122). In this article, we would like to discuss a very specific, contextualized understanding of the above definition. We see the overcoming of one's foregrounds that limit and hinder the development of one's potential as an example of real-life problem-solving. We argue that the skills and attitudes necessary for a successful dealing with such problems and handling the impasses which inevitably occur, can and should be intentionally shaped from the very beginnings of mathematical education.

We start with bringing closer one of the core concepts of this article, i.e. that of *foregrounds*, with a particular emphasis on the notion of a *ruined foreground*. Then we present two psychological approaches we find to be important perspectives on the human being, worth to be reminded and reconsidered by contemporary educators, namely the transgressive concept of [hu]man [being] formulated by Józef Kozielecki and logotherapy proposed by Viktor Frankl. Next, we stress some factors already recognized as standing in the way of the realization of students' potential which deprive them of their dreams and hopes. We discuss the educational objectives, emphasizing the objectives of mathematical education, and argue that one of the most important goals for educators is to enable students to strive for their dreams and transgress their ruined foregrounds. Finally, we state that this mission can be accomplished especially by means of mathematical education. Given that our students can learn mathematics for the purpose of transgressing their personal foregrounds, we need to pay much more attention and make the necessary efforts to provide them from the early years of schooling with a transgressively oriented teaching of this subject, which is the final conclusion of our article.

Foregrounds

Since the term 'foregrounds' may evoke misleading associations, we want to make a clear connection between our understanding and use of this term and the work of Ole Skovsmose (2005, 2012, 2014). According to this author, a person's foreground can be defined as 'the opportunities, which the social, political and cultural situation provides for this person' (Skovsmose, 2005, p. 6). In Skovsmose's words: 'Being born into a certain context makes available a configuration of life opportunities, defined through statistical parameters that signify expectations about length of life, guality of schooling, affluence or poverty, etc.' (2014, p. 5). A foreground is thus a complex structure of external parameters (e.g. life conditions, economic status, length of schooling) combined with subjective factors (e.g. expectations, tendencies, possibilities, hopes), which greatly influence intentional decisions that individual make while choosing their way of personal development and life direction. Foreground includes one's dreams and hopes, but it may also contain a lot of uncertainty, especially when a person finds certain dreams barely reachable. Foreground is best described as a process (Skovsmose, 2014). Since it depends on the external conditions and opportunities, as much as on the internal interpretations of what is possible and to what extent a person has control over life circumstances, it may change over time. When people find their dreams unattainable, they may decide not to fight for them and redirect their attention toward something that seems to be within their reach. In such a case, Skovsmose (2014) speaks about ruined foregrounds which may be one of the most important learning obstacles and reasons for students' failure at school.

We extend this understanding by taking a three-fold perspective on foregrounds. Firstly, we understand 'foregrounds' as a general landscape that is shaped by the aforementioned agents, and this understanding is in line with the definition we recalled. Secondly, we think of a foreground as a horizon line that changes with every step one makes toward it. Wherever we are, there always exists some horizon line we can see in front of us. The third understanding refers to one's foregrounds

as a personal reality consisting of a sequence of steps one takes and decisions one makes in particular 'in the moment' situations. The first understanding may best estimate the feelings and impressions of individuals who are submerged into their life reality. The latter analogies highlight the fact that one's foreground, being out of one's reach, may seem to remain a quite remote perspective, yet, at the same time, it is being created and shaped with every single step one makes. This, again, reflects Skovsmose's understanding of foregrounds as a process. From this perspective we could describe foregrounds as something that at the same time continuously 'is', and discretely 'is happening' in the moment.

We argue that it is important to raise the awareness of teachers about the role they play in shaping students' foregrounds and connect this quite new concept with both the theoretical background teachers have and the school practice that is part of their daily experience.

Human nature from the perspective of the transgressive concept of human beings and logotherapy

During vocational preparation, prospective teachers of mathematics take a few courses on pedagogy and psychology of education. All the numerous theories of the teaching and learning they are introduced to are driven by some philosophical principles. The learned theories together with the inner, personal conceptions held by the teachers strongly influence their priorities, teaching style and the decisions they make in the classroom. Since the belief structures of teachers have already been addressed by many authors, here we only want to emphasize the importance of one of the 'hidden variables' (Leder, Pehkonen, & Törner, 2006), which is a component of that structure, namely the teacher's premises about the nature of a human being. Even if teachers before entering the classroom are not necessarily aware of some of the assumptions they may have, these assumptions are working anyway. For example: teachers who believe that their humanities students can never do mathematics at an advanced level will not encourage them to look for more complicated and complex problems. Instead, they may be telling the students not to expect too much from themselves. Moreover, such teachers may apply a very narrow, one-dimensional framework when interpreting the behaviour of their students. What happens in the classroom often serves the teachers only to confirm their prior beliefs: the difficulties and shortcomings of the students come from the fact that they are not, and will never become, 'math persons'. Beliefs of this kind, if held by a teacher, are never just that teacher's problem. They infect the whole classroom environment and affect the self-perceptions of students. Such teachers unwittingly ruin not only the mathematical foregrounds of their students, because having fixed intelligence mindsets, they divide people into math and non-math groups. Even more damaging to the students is that the teachers do not believe they could ever change.

The human nature has been intriguing researchers for many decades. To describe all of the attempts that have been made thus far in order to understand its phenomenon is beyond the scope of this article. Here, we focus our attention on the contributions of two psychologists that we find particularly enriching and relevant for understanding the nature of the human being. Some psychological theories emphasize the role that childhood experiences play in shaping our adult life. It is believed that unconscious processes originating from early childhood memories can, at least partially, account for our current emotions, attitudes and beliefs. Our backgrounds unquestionably affect our future. It would be, however, far too much to say that they determine who we become. It is especially so in the context of the transgressive concept of human being, which offers a dynamic and empowering view of a person. The term transgression had been defined in different contexts (e.g. geology and genetics) long before a Polish psychologist Józef Kozielecki brought it into the psychological ground. In geology transgression means the spreading of the sea over land; in genetics it denotes a peculiar case of heterosis – the increase in different characters of the hybrids over those of their parents. Kozielecki (1987) uses this term in the context of psychology to speak about overcoming physical, social or symbolic boundaries. According to this author, to

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transgress means to intentionally exceed some boundaries or the limitations of one's current state. The notion of homo transgressivus introduced by Kozielecki (1997) encapsulates the author's vision of human beings who are self-directed, expansive creatures capable of intentionally crossing the boundaries of what they are and what they own, to become who they might be, and to obtain what they might possess. Although environmental and social factors affect the life of homo transgressivus, they do not determine it. Kozielecki states that it is not true that people always take only protective actions and try to maintain the status quo. Sometimes they do just the opposite: they intentionally seek for an arousal that would disrupt the equilibrium and challenge their sense of security. Transgressive actions are purposeful behaviours 'of every kind whose outcomes go beyond the boundaries of past accomplishments and lead to change' (Kozielecki, 1989, p. 45). Such actions are usually driven by a heterostatic motivation, governed by the principle of growth. Adopting this view of the human nature to the educational ground, we no longer see students as completely determined by their life circumstances, prior experiences or achievements. Neither broken backgrounds, nor ruined foregrounds can deprive students of this particularly human capability of transgressing obstacles that they have to face on the way toward their life fulfilment. Theoretically speaking, personal transgressions are within the reach of every person. Not everyone, however, undertakes transgressive actions in order to improve their life situation and solve existential problems. We regard a transgressive orientation toward life as a multidimensional quality that is formed and shaped from the earliest childhood by external social, educational and cultural conditions, as well as internal, individual properties influencing the way people interpret what happens in their lives. People acquire such an orientation in a lingering process. We claim that a large part of the responsibility for inculcating such qualities remains at the educational level. And since solving problems lies at the heart of mathematics, we believe that mathematics teachers are specially predisposed to carry out this mission. Owing to the fact that they can create such a rich learning environment wherein students may undergo many successful personal transgressions, we expect at least some of the life lessons learned in the course of such experiences to be transferred and applied to other domains of students' lives.

According to Viktor Frankl, every human being has a multidimensional nature. The author emphasizes the existence of three dimensions that play an important role in human life: somatic (physical), psychological and noetic (spiritual). He claims that no truth can be found about human if the search of it is restricted to only one of these dimensions. Similarly to Kozielecki, Frankl also challenges the deterministic views of the human nature popularized by behaviour and psychodynamic theories. In Frankl's (1985) words:

[Hu]man is not fully conditioned and determined but rather he [or she] determines himself [or herself] whether he [or she] gives in to conditions or stands up to them. In other words, [hu]man [being] is self-determining. [Hu]man does not simply exist but always decides what his [or her] existence will be, what he [or she] will become in the next moment. (p. 154)

Being free to construct their own characters, people are responsible for who they become. Thus, what matters most are not their current features or instincts, but rather their attitudes and the way in which they approach life circumstances. Reflecting on who they are, human beings are capable of questioning themselves and evaluating and judging their own decisions. Logotherapy proposed by Frankl is a psychotherapeutic approach based on the premise that all humans have free will and, being motivated by the will to meaning, they seek the meaning of life. Referring to Nietzsche's dictum 'He who has a why to live can bear almost any how', Frankl adopts a future-oriented perspective, believing that the human will to meaning always has the last word: regardless of the external conditions, meaning, values and rays of hope can always be found in one's life. However, it is 'only when we redirect our focus from self-interest to something bigger than and beyond ourselves [that we] can experience meaning in life' (Wong, 2014, p. 158). Now, adopting this perspective to the educational ground, we argue that since the lower dimensions distinguished by Frankl are intertwined within a multidimensional complex nature of the human being as such, it is impossible to act on any of them, without

affecting the whole structure. For many years it is the school environment that significantly affects the process of 'becoming a person'. Thus, it is important for educators to know that whatever kind of cognitive, affective or behavioural experiences they provide their students with on the physical and psychological level, they will surely leave some imprints on the noetic level, too. And since this inter-dimensional exchange is always taking place within a structure, whether we focus on it or not, our students could greatly benefit if we began addressing noetic issues purposefully and explicitly in the classroom. For example, a deeper reflection on a mathematical problem that has just been solved in the class, can create an outstanding opportunity to link mathematical activity with issues that go beyond the scope of mathematics. The teachers may ask the students some questions regarding the correctness of the obtained solution, the existence of some other ways of solving the problem, but they can also go further. A reflection on a problem that appears to have more than one possible solution is a good starting point to note that also in the case of real-life problems there might be more than just one way of dealing with them. And only those who make the first step and then go further will ultimately solve the problem. It is important to ask students what keeps them from making the first or further steps, what they have learned from solving a particular problem, and finally, what they have learned about themselves while solving it. Students who find value in seeing such a connection between mathematics and their personal development may become accustomed to the process of going back and forth between mathematical problems and real-life situations. This new habit of mind may contribute greatly to the transferring of what students have learned in the classroom – about who they are, how they react when facing a problem, and what strategies of solving problems they have discovered to be effective - into real life situations, especially when they are about to face some serious problems.

There is no doubt that ruined foregrounds are one of the most difficult existential problems of humankind. They are burning issues people should deal with, yet at the same time, when trying to face them, they too easily give up and lose hope. Adopting the perspective of the transgressive concept of human and being profoundly convinced that especially mathematical education can be used as the means toward multidimensional personal development, we argue that by frequently repeated valuing it is possible to instil in students a habit of transgressing the obstacles and challenging the limitations they encounter. If we expect such an orientation to permeate the human way of being, it should seep through one's various experiences and be inculcated from early childhood. To get a deeper understanding of what the individuals' ruined foreground could mean, we shall now elaborate on the factors that make some life dreams and hopes unattainable to them.

The landscape of ruined foregrounds

Among the mathematics-related factors potentially contributing to the ruining of students' foregrounds, it is important to mention external factors, like the social status of the scientific discipline of mathematics, and the social perception of mathematics – the school subject, the socio-economic status of a student's family and the educational level of one's parents, as well as some personal agents, with the crucial role of a student's self-esteem, self-efficacy and sense of being in control over one's life experiences.

Undoubtedly, since governments and policies often put mathematics in the position of a stepping-stone to further stages of education (e.g. Vinner, 2013), mathematical knowledge and skills are the means for better educational and job opportunities. Martin (1986, p. 13) recognizes school mathematics to be 'the principal filter of the education system'. Given that students are labelled as 'maths' or 'non-maths', even at the early stages of their school career, mathematics is considered a great factor contributing to school and academic failure. Moreover, as noted by Volmink (1994):

Mathematics is not only an impenetrable mystery to many, but has also, more than any other subject, been cast in the role as an 'objective' judge, in order to decide who in the society 'can' and who 'cannot'. It therefore serves as the gate keeper to participation in the decision making processes of society. To deny some access to participation in mathematics is then also to determine, a priori, who will move ahead and who will stay behind. (pp. 51–52)

Although selections seem to be an inherent part of the educational system rather than mathematics and thus are not something that mathematics should or could be blamed for, it inevitably contributes to the often negative perception of mathematics in society. Negative emotions it evokes, which lead to the disease of mathophobia (also known as math anxiety), preclude the development of the potential of many students.

Since the publication of the Coleman Report (1966), socio-economic status has been considered an important factor shaping and explaining the differences in the educational attainment of students. Economic status is a factor that differentiates children already at the beginning of school education and explains the differences in tests better than school grades (Gajda, 2015; Sackett, Kuncel, Arneson, Cooper, & Waters, 2009). Variables describing the financial situation of the family, like the material and home resources (e.g. car, phone, computer, household appliances) are also a relatively strong predictor of mathematical skills. The weakest effect was observed for the index of cultural capital, but it was still significant (e.g. the number of books in the house; Kaczan & Rycielski, 2013). The Faure Report (1972) highlighted the existence of a close relationship between socio-cultural handicaps and parental behaviour, including attitudes linked to social environment as well as personal factors. Educational differences among students with various socio-economic backgrounds tend to deepen with age (Caro, McDonald, & Willms, 2009; Condron, 2007) and have a long-lasting impact on the educational attainment and the position in the labour market (Alexander, Entwisle, & Olson, 2007; Kerckhoff, Raudenbush, & Glennie, 2001). The gap among children can be observed very early: pre-school children from families with low socio-economic status already perform worse on mathematical tasks than their middleclass peers (Jordan, Huttenlocher, & Levine, 1994). Such children use less adequate strategies when coping with difficult situations, such as solving mathematical problems (Jordan, Kaplan, Ramineni, & Locuniak, 2009). It shows how significant impact life conditions and economic status have on the level of children education.

As can be seen, school successes of children depend much on their living conditions. Having poor life perspectives means having a difficult educational and life start. On the other hand, however, a socio-economic success depends largely on the level of mathematical achievements. Good mathematical education is a source of numerous benefits to individuals, such as the acquisition of logical and critical thinking skills which strengthen their social position. Many researchers point out that high mathematical skills have a large impact on achieving success in life (Rivera-Batiz, 1992). It also influences the choice of health care (Reyna, Nelson, Han, & Dieckmann, 2009), as well as retirement decisions and is correlated with salary (Dougherty, 2003). Poor mathematical achievements have a negative impact on employment opportunities, even to a greater extent than reading difficulties (Bynner & Parsons, 1997). There is evidence that mathematical illiteracy is associated with significant costs which the whole society has to bear (Report of the Organization for Economic Co-operation and Development, 2010). It is well known that it is hard to change the public system generally, but there are indications enabling us to conclude that perhaps it is possible to improve the living conditions within the society by raising the educational level of its members.

Among different measures of family resources, the educational level of parents is the strongest predictor of children's school achievements. The educational level of parents and the economic and cultural capital of the family are more closely related to mathematical skills than reading or writing (Kaczan & Rycielski, 2013). A higher family status also affects parental decisions regarding child education planning. Wealthy parents care more than the less affluent ones about choosing a school that will assure their child better educational outcomes in the future. As a result, students born in families with a high socio-economic status usually go to better schools and ultimately receive better educational opportunities, even if their achievements at the initial stage of education are not outstanding (Dolata & Jarnutowska, 2012). Also, well-educated parents are aware of the importance of education, so it is not surprising that their aspirations play a significant role in influencing the achievements of their children. Such parents have positive attitudes toward education and

they engage more closely in the upbringing of their children and help them do their homework. Home environment consists of different motivational variables, but parental involvement is considered the most important. The attitudes of children towards mathematics depend on the general home environment and math attitudes of their parents (Soni & Kumari, 2015). As shown by Jackson (2008), negative emotions related to mathematics can be transferred to children via the beliefs of their parents. Adults' false beliefs about mathematics and the nature of the learning of mathematics negatively influence mathematical attitudes are inborn, thus in order to be good in mathematics one needs to have a special talent or predisposition toward it (see also: Kloosterman & Stage, 1992). Such beliefs cannot be formed in a child by themselves. They are absorbed from the socio-cultural messages in the process of acculturation.

The early years of school education are the most vulnerable time for learning mathematics. The very first experience of mathematics forms the foundations for a future development of mathematical cognitive skills and affective orientation toward mathematics. Children who do not have problems with mathematics at the beginning of their education often develop their skills and affects normally over time, whereas kids who had mathematical difficulties at the beginning, are getting worse with time (Ramani & Siegler, 2011). If not resolved at the early years of education, mathematics-related difficulties, both of the cognitive and affective nature, deepen in the course of education. Mathematical failures negatively affect the self-assessment of one's mathematical abilities. This leads to a decrease in motivation – students who experience learning problems tend to avoid mathematics (Hadfield & Lillibridge, 1991).

Recurring failures and avoidance also affect the self-esteem of individuals (Wigfield, Eccles, Mac Iver, Reuman, & Midgely, 1991). Children who cannot solve math problems, compare themselves with their successful colleagues. Sometimes it is also the teacher who inadvertently causes a lot of harm by making comparisons in the classroom. The sense of being 'not good enough' to succeed is easily generalized by children. Oftentimes, they come to believe that they will be permanently performing worse than others. A commonly held belief, widely spread in the society, that high mathematical achievements are the direct indicators of the level of intelligence, gives rise to inferiority complexes. Children with a fixed mindset who feel stupid or less intelligent, activate various defence mechanisms in order to protect themselves from the occurring tension and uneasiness they can barely handle. They will then tend to avoid mathematical activity, hesitate to speak out loud during the lesson so as not to be in the spotlight of their peers' attention, and pretend that they know how to do math problems. Students who are convinced that they are not capable of achieving certain goals (e.g. solving certain kind of math problems), easily refrain from making further efforts. Low self-efficacy finally deepens and consolidates the educational difficulties and is a strong predictor of mathematics performance, even stronger than math anxiety or previous math experiences (Ayotola & Adedeji, 2009; Pajares & Graham, 1999). Numerous educational failures, especially when taking place at a very early stage, can shape children's beliefs regarding the control they have over their mathematical achievements. Due to the fact that mathematics has a very high social status, failures in this field cause a lot of damage to one's self-image. Since this is a very vulnerable area, beliefs formed in relation to mathematics spread over one's whole life. In particular, the feeling of not being in control in the field of mathematics may turn into the feeling of not being in control of one's life experiences.

The above set of mathematics-related factors shows clearly that students' foregrounds, understood as life opportunities, may be ruined because of the social status and elite approach toward mathematics; social, economical, cultural and educational backgrounds which stand in one's way toward the development of mathematical competencies, and negative experiences of learning mathematics, which widely infect one's self-image. Ruined foregrounds as pictured above resemble a battleground at a recess after a battle of two disproportionally equipped armies. A vast majority of the equipment that young children and older students should be provided with to win that battle ought to come from school education, and we believe that – due to its specific nature – a large part of this responsibility rests with mathematical education.

Mathematics education - a twofold sword

Among the many scientific disciplines transposed into school subjects, mathematics has a very specific status in relation to the process of foregrounding. On the one hand, as has been shown, it may ruin students' dreams and hopes about their further education and position in the labour market. On the other hand, however, it may be used to rebuild at least some of the ruined foregrounds. In that sense, we say it is a twofold sword, which can do a lot of harm, as much as a lot of good, depending on how it is going to be used.

According to Dolk and te Selle (2010), 'education serves three main purposes: it adds to the personal development of children, it supports the creation of social and cultural awareness among children and it prepares children for their participatory role in society' (p. 15). In particular, mathematics is said to enrich students through at least three kinds of empowerment it gives (Ernest, 2002), namely: mathematical – the ability to use the subject matter knowledge and skills in school mathematics, social – the capacity to use mathematics for a better functioning within the society, and epistemological – related to one's personal confidence, mathematical self-efficacy and power over mathematical knowledge. Researchers have also been considering issues like, for instance, fairness, equity and social justice (e.g. Gutstein, 2003; Simic-Muller, 2015) in relation to the curriculum, textbooks content and the practice of mathematics teaching. It is believed that mathematicians share the responsibility for taking up vital issues concerning society, such as peace, politics, gender equity, tolerance and so forth. Thus, since the mathematics classroom may be a place where contemporary problems of humankind are being addressed and discussed, it is not an exaggeration to say that it is worth teaching and learning mathematics for the sake of humanity.

In recent years, the inclusion of such values in the educational process has become an urgent problem in the scientific world and, above all, a challenge to pedagogical practices. Formulated learning objectives, regardless of the current education system, refer to values considered important by the society and these values are usually closely related to a specific perception of the human nature. Among the many approaches to the objectives of mathematics education, one seems to be exceptionally adequate and still valid.

Already in the 1980s, Krygowska (1986) argued that mathematics education should include both its proximal and further objectives. She formulated three levels of objectives which, despite the laps of time, remain a point of reference to many teachers of mathematics and researchers from the field of mathematics education. Level I contains basic mathematical knowledge and skills, usually described in the curriculum, formulated as more or less operationalized results (e.g. students know ..., students know how to define ... and so forth). Level II refers to attitudes and behaviours specific to mathematical activity and mathematical methodology. Students should have, for instance, an active attitude toward mathematical problems, some disposal to perceive and formulate problems in familiar contexts, the skill of using simple strategies while solving a problem and the understanding of the meaning of definition and the meaning of proof. Level III refers to attitudes and intellectual behaviours being transferred beyond the domain of mathematical activity. In that sense, it is assumed that the meaning of teaching mathematics as part of general education consists in, among others, the intellectualization of attitudes and behaviours of the wide strata of the society. On the assumption that the objectives of Level II and III are realized almost automatically, with no need for a special assist of the teachers, rarely are mathematics curricula formulated with a deep reflection on these levels. With respect to students' foregrounding, we argue that especially the objectives of the third level, recognized by Krygowska as those with the utmost importance, need to be paid more attention to. Since they go far beyond the scope of mere school knowledge and skills, they may play a significant role in shaping the most important life competencies needed for a successful life and sustainable development.

Problem-solving as a key life competency

In the last decades, it was the problem-solving that has been recognized as one of the key competencies people need in order to successfully deal with the challenges of the contemporary world. Many efforts have been made by educational policy-makers to make the school a place that helps to develop and foster these competencies from the early years of education. Since there is a general agreement that problem-solving lies at the heart of mathematics as a scientific discipline, it is not surprising that it has become one of the main priorities and learning goals included in many mathematics curricula worldwide.

Already in the year 1980, National Council of Teachers of Mathematics (NCTM) released eight recommendations for school mathematics. The first among the listed recommendations that was explicitly formulated there says that 'problem solving must be the focus of school mathematics' (p. 2). According to this document:

Problem solving involves applying mathematics to the real world, serving the theory and practice of current and emerging sciences, and resolving issues that extend the frontiers of the mathematical sciences themselves. (...) True problem solving power requires a wide repertoire of knowledge, not only of particular skills and concepts but also of the relationships among them and the fundamental principles that unify them. (NCTM, 1980, p. 2)

Also the OECD advocates for promoting problem-solving and defines this competency as 'an individual's capacity to engage in cognitive processing to understand and resolve problem situations where a method of solution is not immediately obvious. It includes the willingness to engage with such situations in order to achieve one's potential' (OECD, 2013, p. 122).

Common understanding of a problem usually encompasses the given state, the desired goal and the obstacles that stand in the way toward achieving it (Mayer, 1992). According to the literature on mathematics and methods of teaching it, there are at least six important features of what mathematicians consider a problem:

- Affective and intellectual engagement, 'the risks and rewards concomitant with that commitment' (Schoenfeld, 1983, p. 41) together with a strong orientation toward obtaining the solution: a problem is not a problem until someone wants to solve it.
- Lack of already known procedures one could follow in order to solve the task which in itself does
 not have to be exceptionally tricky it is not the difficulty that constitutes a problem, it is the
 inability of a person to come to grips with an issue straight away that does (Ciosek, 2005). Thus
 solving a problem is transgressive in its nature, for it requires going beyond one's knowledge,
 accessible routine schemas and ordinary thinking.
- Relative sense of difficulty and ambiguity: what is a problem to one person, may not be such to
 another. There are, however, some problems, which are difficult to the whole community of mathematicians and sometimes it takes centuries before the solutions are found.
- Inefficacy of the first attempt of solving, which is often followed up by increasingly strong emotional responses.
- Intellectual rather than computational (Schoenfeld, 1985, p. 74) and affective difficulties associated with the task that challenge the problem solver. To solve a problem means to overcome these obstacles. Again, it is clear, that problem-solving bears the characteristics of transgressive actions (of cognitive and affective nature).
- Solving a task that is a problem enriches the individual (the community, the whole nation or even humankind).

Dossey (2017) notes that while the word problem alone has negative connotations and is typically matched with difficulty associated with tension and unease, the word solving brings in the relief. Regarding mathematics education, the term problem-solving refers undoubtedly to tasks (a) being an intellectual and emotional challenge to pupils, (b) promoting students' conceptual understanding

of the subject matter and (c) developing students' mathematical and mathematics-related competencies like, among others, reasoning, logical thinking, communicating their arguments for/against some ideas.

Naturally, if mathematics classes are to develop a problem-solving competency in students, problem-solving has to become an integral part of the learning of mathematics. This postulate is typically realized either by teaching problem-solving or by teaching through problem-solving from the early years of schooling. There is no doubt both of these activities are valuable and contribute greatly to the cognitive and affective development of students. Teaching problem-solving focuses on providing students with some useful strategies recognized as helpful in the process of problem-solving. They may be related to mathematics, or may be universal. Famous mathematicians like, for example, Jacques Hadamard, Henri Poincairé and George Polya were eager to share their personal experiences with mathematical struggles, as well as ways of thinking they themselves found effective. Now, these essential elements of the mathematical craft are implemented in the classroom and they make students more familiar with the practice of mathematical inquiry. The famous book written by George Polya (1945), a Hungarian mathematician of the twentieth century, entitled How to solve it? provides the reader with four well known (thus just mentioned here) principles of mathematical problem-solving: understand the problem, devise a plan, carry out the plan and look back.

The alternative approach, that is of teaching through problem-solving, advocates building classroom activities around problems (Schroeder & Lester, 1989) related to mathematical concepts, theorems and ideas. They are proposed accordingly to particular topics included in the curriculum. This approach provides students with many opportunities to solve problems. Polya (1962), when considering problem-solving, wrote:

[it is] a practical art, like swimming, or skiing, or playing the piano. (...) If you wish to learn swimming you have to go into the water and if you wish to become a problem solver you have to solve problems. (p. V)

No matter which of the above approaches the teachers adopt, it is very likely that they would often try to design situations where 'children might learn, by becoming apprentice mathematicians, to do what master mathematicians and scientists do in their everyday practice' (Lave, Smith, & Butler, 1988, p. 62). This is somewhat problematic and we would like to briefly comment on this. Perhaps the gist of our concern was shared by Hiebert et al. (1996), who wrote the following words:

the metaphor of children as small mathematicians can be pushed too far. Children are different than mathematicians in their experiences, immediate ambitions, cognitive processing power, representational tools, and so on. If these differences are minimized or ignored, children can be thought of as small adults and education can become a matter of training children to think and behave like older adults. (...) From our perspective, children need not be asked to think like mathematicians but rather to think like children about problems and ideas that are mathematically fertile. (p. 19)

The authors advocate for promoting mathematical inquiry instead of mastering the rules, imitating strategies, taking full advantage of the logic of the subject instead of relying basically on the teacher and her support. In order to achieve this, however, children need to be allowed to make mathematics problematic, by which the authors mean: 'allowing students to wonder why things are, to inquire, to search for solutions, and to resolve incongruities. It means that both curriculum and instruction should begin with problems, dilemmas, and questions for students' (p. 12).

If we expect the teaching of mathematics to have a significant impact on students' proximal and further foregrounding, it must take into account, without fail, on the one hand – who the students are today, and on the other – who they are to become tomorrow. It is, however, deficient from the perspective of their development to provide them with mathematically rich environments. Such environments need to be made *accessible* to all the students, so that they could dive into the problems and explore new mathematical lands with the teachers' assistance. What is even more important in terms of students' motivation is that in order to become explorers and investigators, they have to find the activities we invite them to participate in personally relevant and meaningful.

Transgressing personal foregrounds through the means of the learning of mathematics

According to the Faure Report (1972): 'Every educational act is part of a process directed towards an end' (p. 145). It means that both the teaching and the learning of mathematics are transgressive in their nature. However, if mathematics education is to go beyond the school walls and change the real lives of students, we need to have teachers who would cross the boundaries of the subject and purposefully address issues that go beyond the scope of mathematics. We argue that teachers of mathematics should make explicit connections between mathematical problem-solving and real-life problems that hinder one's potential and limit one's foregrounding. We pay a lot of attention to problem-solving, for it is of the utmost importance for us to emphasize that many of the skills and attitudes we have in our adult life originate from the early childhood experiences. And one of the attitudes that is being shaped from the early years is the attitude we have toward life problems. The skill of coping with difficulties can be fostered, especially in the math classroom, by experiences of previous effective struggles. The best way to exercise such skills is to teach students, as early as possible, how to confront problems in a constructive manner. Students could benefit to a great extent from finding in the learning of mathematics meanings relevant to their personal lives.

It is well known that young people today seek mentors, leaders and motivational teachers. They want to know not only where to go and how to get there, but also why they should care. Oftentimes, some students ask 'Why should I learn this?'. We interpret such a question as an exemplification of what Frankl (1985) called 'the will to meaning'. Such a question is undoubtedly a great challenge for the teacher. Even among the researchers within the field of mathematics education there is no clear agreement on what could be the best possible answer (e.g. Dudley, 2011; Ernest, 2010; Lockhart, 2009; Vinner, 2013; Wu, 1997). However, if we look into the reasons mathematicians have for doing what they do, it becomes clear that they have their whys. Problem-solving 'lies at the heart of mathematics' (Cockcroft, 1982), and what often constitutes the core of the problem-solving process is the search for meaning, looking for patterns to be uncovered and described. Great part of the efforts made by mathematicians toward finding the solutions is driven by the desire to take control over the chaos, unpredictability and continuously changing conditions. By uncovering the patterns and describing them, a human being is capable of subjugating at least some part of the surrounding reality. In Schoenfeld's (1983) words: 'The value to working the problem lies in the solution process. By making systematic observations of a "messy" phenomenon, one gains insights into its nature' (p. 41). If teachers want to inspire students and evoke their interest in problem-solving, they should show the essence of being a professional problem solver: 'what makes a mathematician is not technical skill [hows] or encyclopedic knowledge [whats] but insatiable curiosity and a desire for simple beauty [whys]' (Lockhart, 2014, p. 10). We advocate for school mathematics to focus less on whats and hows, and more on whys standing behind mathematical engagement.

It also happens quite often that professional mathematicians appreciate their discipline for being 'useless' in terms of practical daily utility (Lockhart, 2009). Some mathematicians simply like the fact that in their imagination they can think of any object they want and then play with it, experiencing a great deal of curiosity and even greater load of fun. Paul Lockhart is an example of a mathematician of that kind. In his famous essay, *A Mathematician's Lament*, the author explicitly states that:

It would be bad enough if the culture were merely ignorant of mathematics, but what is far worse is that people actually think they *do* (emphasis in the original) know what math is about – and are apparently under the gross misconception that mathematics is somehow useful to society! This is already a huge difference between mathematics and the other arts. Mathematics is viewed by the culture as some sort of tool for science and technology. Everyone knows that poetry and music are for pure enjoyment and for uplifting and ennobling the human spirit (hence their virtual elimination from the public school curriculum) but no, math is important (emphasis in the original) (Lockhart, 2009, p. 6)

Teachers often tell their students they should learn mathematics, because it is everywhere they go, it has plenty of applications and it is the gateway to further education and better job opportunities.

Unexpectedly, imposing the utilitarian perspective onto our students may do more harm than good, and eventually contribute to the ruining of some foregrounds. Keeping this in mind, we argue for the shift of educators' attention from the applications of mathematics to making use of mathematics in the context of what is personally meaningful to an individual.

Another important aspect of doing mathematics is that in the mathematics classrooms students often might get the impression that each problem has an easily obtainable solution. Such experiences contribute to the formation of beliefs that are a real threat to students' self-esteem and further engagement. Students begin to believe (Kloosterman & Stage, 1992; Schoenfeld, 1992) that solving a problem should take no longer than five minutes (that is what they see in the classroom) and that if they do not know how to solve a problem right away, they will not solve it at all. After all, the problems they were exposed to in the classroom always had easy solutions. Kloosterman and Stage (1992) suggest that good counter-examples might contribute to preventing as well as changing these misleading and maladaptive beliefs. For example, students who think they are not capable of solving a task when it requires much time, should experience success in solving a time-consuming problem to see that their previous convictions were wrong. Many real-life problems require a lot of time and patience. Oftentimes only those who do not let themselves be discouraged by the complexity and difficulty inherent in the problem, ultimately obtain what they have been striving for.

Pupils should also see and experience mathematics in a similar way to how mathematicians do it:

Mathematical reality is an infinite jungle full of enchanting mysteries, but the jungle does not give up its secrets easily. Be prepared to struggle, both intellectually and creatively. (...) The important thing is not to be afraid. So you try some crazy idea, and it doesn't work. That puts you in some pretty good company! Archimedes, Gauss, you and I – we're all groping our way through mathematical reality, trying to understand what is going on, making guesses, trying out ideas, mostly failing. And then every once in a while, you succeed ... And that feeling of unlocking an eternal mystery is what keeps you going back to the jungle to get scratched up all over again. (Lockhart, 2014, pp. 2, 15)

Early disappointments and experiences of one's lack of efficiency may contribute greatly to the negative self-perception. Especially young students coping with failures easily lose hope for a better future. But what if they could learn that getting stuck is a rule rather than an exception in mathematics (and so is failure)? As long as getting stuck and failing despite some efforts are interpreted by students as clear signs of incapability, we will have many students discouraged from entering 'the jungle', which means many students with ruined foregrounds.

Conclusion

Every problem is a learning opportunity. There are many things a person can experience while being submerged in the mathematical struggle. One can learn, for example, that a problem will not always be solved. While solving problems, students may also learn that sometimes they will not understand everything immediately, but there is always a lot to be discovered. Not giving up on hope and struggling are good opportunities to exercise strong will and persistence. While solving a problem we can also learn a lot about who we are, what our reactions are like, how we behave when facing a problem, whether we are strong enough to handle the tension and uncertainty and so forth. Difficulty arises when we do not accept what we find out about who we are. If it happens so, we are invited to work on what does not work for us, having as much understanding and patience as possible. Maybe this is what Piaget (1973) meant describing mathematics in the following way: 'there is no field [other than mathematics] where the "full development of the human personality" and the mastery of the tools of logic and reason which insure full intellectual independence are more capable of realization' (p. 99). And this is the teachers' role to make this specific use of mathematics in the service of personal growth visible to students. If the teaching of mathematics is transgressively oriented and addresses issues that go beyond mere

subject matter knowledge, then students can use the competencies they obtain while learning mathematics in order to transgress their personal foregrounds.

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References

Alexander, K. L., Entwisle, D. R., & Olson, L. S. (2007). Lasting consequences of the summer learning gap. American Sociological Review, 72, 167–180.

Ayotola, A., & Adedeji, T. (2009). The relationship between mathematics self-efficacy and achievement in mathematics. Procedia – Social and Behavioral Science, 1(1), 953–957.

Bynner, J., & Parsons, S. (1997). Does numeracy matter? Evidence from the national child development study on the impact of poor numeracy on adult life. London: The Basic Skills Agency.

Caro, D. H., McDonald, J. T., & Willms, J. D. (2009). Socio-economic status and academic achievement trajectories from childhood to adolescence. Canadian Journal of Education/Revue Canadienne de L'éducation, 32(3), 558–590.

Ciosek, M. (2005). Proces rozwiązywania zadania na różnych poziomach wiedzy i doświadczenia matematycznego. Kraków: Wydawnictwo Naukowe Akademii Pedagogicznej.

Cockcroft, W. H. (1982). Mathematics counts. London: HM Stationery Office.

Coleman, J. J., Campbell, E. Q., Hobson, C. J., McPartland, J., Mood, A. M., & Weinfeld, F. D. (1966). Equality of educational opportunity. Washington, DC: U.S. Government Printing Office.

Condron, D. J. (2007). Stratification and educational sorting: Explaining ascriptive inequalities in early childhood reading group placement. Social Problems, 54(1), 139–160.

Dolata, R., & Jarnutowska, E. (2012). Status społeczny rodziny pochodzenia ucznia i jego korelaty a wyniki nauczania. In Szkolne, psychologiczne i społeczne uwarunkowania wyników nauczania na I etapie edukacyjnym (pp. 240–291). Warszawa: Instytut Badań Edukacyjnych.

Dolk, M., & te Selle, A. (2010). Ideas for natural differentiation in primary mathematics. Rzeszów: Wydawnictwo Uniwersytetu Rzeszowskiego.

Dossey, J. A. (2017). Problem solving from a mathematical standpoint. Educational research and innovation. In B. Csapó & J. Funke (Eds.), The nature of problem solving: Using research to inspire 21st century learning (pp. 59–72). Paris: OECD.

Dougherty, C. (2003). Numeracy, literacy, and earnings: Evidence from the national longitudinal survey of youth. Economics of Education Review, 22(5), 511–521.

Dudley, U. (2011). What is mathematics for? In M. Pitici (Ed.), *The best writing on mathematics 2011* (pp. 1–11). Princeton, NJ: Princeton University Press.

Ernest, P. (2002). Empowerment in mathematics education. *Philosophy of Mathematics Education Journal*, *15*(1), 1–16. Ernest, P. (2010). Add it up: Why teach mathematics? *Professional Educator*, *9*(2), 44–47.

Faure, E., Herrera, F., Kaddoura, A.-R., Lopes, H., Petrovsky, A. V., Rahnema, M., & Ward, F. C. (1972). *Learning to be. The world of education today and tomorrow*. Paris: Offset Aubin.

Frankl, V. E. (1985). Man's search for meaning. New York, NY: Simon and Schuster.

- Gajda, A. (2015). Osiągnięcia szkolne i kreatywność w poszukiwaniu przyczyn zróżnicowania relacji. Teraźniejszość Człowiek Edukacja, 2(70), 55–78.
- Gutstein, E. (2003). Teaching and learning mathematics for social justice in an urban, Latino school. *Journal for Research in Mathematics Education*, 34(1), 37–73.
- Hadfield, O. D., & Lillibridge, F. (1991). A hands-on approach to the improvement of rural elementary teacher confidence in science and mathematics (Eric Doc. Reproduc. Ser. No. ED 334082). Nashville, TN: Annual National Rural Small Schools Conference.

Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K., Human, P., Murray, H., ... Wearne, D. (1996). Problem solving as a basis for reform in curriculum and instruction: The case of mathematics. *Educational Researcher*, 25(4), 12–21.

Jackson, E. (2008). Mathematics anxiety in student teachers. *Practitioner Research in Higher Education*, 2(1), 36–42.

- Jordan, N. C., Huttenlocher, J., & Levine, S. C. (1994). Assessing early arithmetic abilities: Effects of verbal and nonverbal response types on the calculation performance of middle-and low-income children. *Learning and Individual Differences*, 6(4), 413–432.
- Jordan, N. C., Kaplan, D., Ramineni, C., & Locuniak, M. N. (2009). Early math matters: Kindergarten number competence and later mathematics outcomes. *Developmental Psychology*, 45(3), 850–867.
- Kaczan, R., & Rycielski, P. (2013). Zasoby rodziny a umiejętności szkolne dzieci 6- i 7-letnich. Studia Edukacyjne, 27, 159– 171.
- Kerckhoff, A. C., Raudenbush, S. W., & Glennie, E. J. (2001). Education, cognitive skill, and labor force outcomes. Sociology of Education, 74(1), 1–24.
- Kloosterman, P., & Stage, F. K. (1992). Measuring beliefs about mathematical problem solving. School Science and Mathematics, 92(3), 109–115.
- Kozielecki, J. (1987). Koncepcja transgresyjna człowieka [Transgressive concept of a man]. Warszawa: PWN.
- Kozielecki, J. (1989). Towards a theory of transgressive decision making: Reaching beyond everyday life. Acta Psychologica, 70(1), 43–63.
- Kozielecki, J. (1997). Transgresja i kultura [Transgression and culture]. Warszawa: Wydawnictwo Akademickie Żak.
- Krygowska, Z. (1986). Elementy aktywności matematycznej, które powinny odgrywać znaczącą rolę w matematyce dla wszystkich. Roczniki Polskiego Towarzystwa Matematycznego. Seria V: Dydaktyka Matematyki, 6, 25–41.
- Lave, J., Smith, S., & Butler, M. (1988). Problem solving as an everyday practice, In J. Lave, J. G. Greeno, A. Schoenfeld, S. Smith, & M. Butler (Eds.), *Learning mathematical problem solving* (Institute for Research on Learning Report No. IRL88-0006). Palo Alto, CA.
- Leder, G. C., Pehkonen, E., & Törner, G. (Eds.). (2006). Beliefs: A hidden variable in mathematics education? Dordrecht: Kluwer Academic.
- Lockhart, P. (2009). A mathematician's lament. New York, NY: Bellevue Literary Press.
- Lockhart, P. (2014). Measurement. Cambridge, MA: Belknap Press of Harvard University Press.
- Martin, J. C. (1986). A necessary renewal of mathematics education. In P. Damerow, M. E. Dunkley, B. F. Nebres, & B. Werry (Eds.), *Mathematics for All* (Science and Technology Education Document Series No. 20., pp. 13–17). Paris: UNESCO.

Mayer, R. E. (1992). Thinking, problem solving, cognition. New York, NY: WH Freeman/Times Books/Henry Holt.

- National Council of Teachers of Mathematics. (1980). An agenda for action: Recommendations for school mathematics of the 1980s. Reston, VI: Author.
- Organisation for Economic Co-operation and Development. (2010). PISA 2009 results: What makes a school successful? Resources, policies and practices (t. 4). Paris: Author.
- Organisation for Economic Co-operation and Development. (2013). PISA 2012 assessment and analytical framework: Mathematics, reading, science, problem solving and financial literacy. Paris: Author.
- Pajares, F., & Graham, L. (1999). Self-efficacy, motivation constructs, and mathematics performance of entering middle school students. Contemporary Educational Psychology, 24, 124–139.
- Piaget, J. (1973). To understand is to invent: The future of education. New York, NY: The Viking Press.
- Polya, G. (1945). How to solve It. Princeton, NJ: Princeton University Press.
- Polya, G. (1962). Mathematical discovery: 2: On understanding, learning, and teaching problem solving. New York, NY: Wiley. Ramani, G. B., & Siegler, R. S. (2011). Reducing the gap in numerical knowledge between low- and middle-income pre-

schoolers. Journal of Applied Developmental Psychology, 32, 146–159.

Reyna, V. F., Nelson, W. L., Han, P. K., & Dieckmann, N. F. (2009). How numeracy influences risk comprehension and medical decision making. *Psychological Bulletin*, 135, 943–973.

Rivera-Batiz, F. L. (1992). Quantitative literacy and the likelihood of employment among young adults in the United States. Journal of Human Resources, 27, 313–328.

- Sackett, P. R., Kuncel, N. R., Arneson, J. J., Cooper, S. R., & Waters, S. D. (2009). Does socioeconomic status explain the relationship between admissions tests and post-secondary academic performance? *Psychological Bulletin*, 135 (1), 1–22.
- Schoenfeld, A. H. (1983). The wild, wild, wild, wild world of problem solving (A review of sorts). For the Learning of Mathematics, 3(3), 40–47.
- Schoenfeld, A. H. (1985). Mathematical problem solving. London: Academic Press.

- Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition and sense-making in mathematics. In D. Grouws (Ed.), Handbook for research on mathematics teaching and learning (pp. 334–370). New York, NY: MacMillan.
- Schroeder, T., & Lester, F. (1989). Developing understanding in mathematics via problem solving. In P. R. Trafton (Ed.), New directions for elementary school mathematics. Yearbook, national council of teachers of mathematics (pp. 31–42). Reston, VA: NCTM.
- Simic-Muller, K. (2015). Social justice and proportional reasoning. Mathematics Teaching in the Middle School, 21(3), 163– 168.
- Skovsmose, O. (2005). Foregrounds and politics of learning obstacles. For the Learning of Mathematics, 25(1), 4-10.
- Skovsmose, O. (2012). Students' foregrounds: Hope, despair, uncertainty. *Pythagoras*, 33(2), 1–8.
- Skovsmose, O. (2014). Foregrounds: Opaque stories about learning. Rotterdam: Sense Publishers.
- Soni, A., & Kumari, S. (2015). The role of parental math attitude in their children math achievement. *International Journal of Applied Sociology*, *5*(4), 159–163.
- Vinner, S. (2013). The mathematics teacher: Between solving equations and the meaning of it all. *The Journal of Mathematical Behavior*, 32(3), 474–480.
- Volmink, J. (1994). Mathematics by all. In E. Lerman (Ed.), *Cultural perspectives on the mathematics classroom* (pp. 51–67). Dordrecht: Kluwer.
- Wigfield, A., Eccles, J. S., Mac Iver, D., Reuman, D. A., & Midgely, C. (1991). Transition during early adolescence: Changes in children's domain-specific self-perceptions and general self-esteem across the transition to junior high school. *Developmental Psychology*, 27(4), 552–565.
- Wong, P. T. P. (2014). Viktor Frankl's meaning seeking model and positive psychology. In A. Batthyany & P. Russo-Netzer (Eds.), *Meaning in existential and positive psychology* (pp. 149–184). New York, NY: Springer.
- Wu, H. (1997). The mathematics education reform: Why you should be concerned and what you can do. American Mathematical Monthly, 104(10), 946–954.